CHAPTER 6

Dimensional Analysis and Similitude

6.1 $\frac{g}{V_1^2}\left[\frac{V_1^2}{2g}+\frac{p_1}{rg}+z_1\right]=\frac{g}{V_1^2}\left[\frac{V_2^2}{2g}+\frac{p_2}{rg}+z_2\right].$

$$\frac{1}{2}+\frac{p_1}{rV_1^2}+\frac{gz_1}{V_1^2}=\frac{1}{2}\frac{V_2^2}{V_1^2}+\frac{p_2}{rV_1^2}+\frac{gz_2}{V_1^2}.$$

or $$\underline{\frac{1}{2}+\frac{p_1}{rV_1^2}+\frac{gz_1}{V_1^2}=\left(\frac{1}{2}+\frac{p_2}{rV_2^2}+\frac{gz_2}{V_2^2}\right)\frac{V_2^2}{V_1^2}}$$

6.2 a) $[\dot{m}]=\frac{\text{kg}}{\text{s}}=\frac{\text{N}\cdot\text{s}^2}{\text{m}\cdot\text{s}}=\frac{\text{N}\cdot\text{s}}{\text{m}}. \quad \therefore \frac{FT}{L}.$

b) $[p]=\frac{\text{N}}{\text{m}^2}. \quad \therefore \frac{F}{L^2}$

c) $[r]=\frac{\text{kg}}{\text{m}^3}=\frac{\text{N}\cdot\text{s}^2}{\text{m}\cdot\text{m}^3}=\frac{\text{N}\cdot\text{s}^2}{\text{m}^4}. \quad \therefore \frac{FT^2}{L^4}.$

d) $[m]=\frac{\text{N}\cdot\text{s}}{\text{m}^2}. \quad \therefore \frac{FT}{L^2}$

e) $[W]=\text{N}\cdot\text{m}. \quad \therefore FL$

f) $[\dot{W}]=\frac{\text{N}\cdot\text{m}}{\text{s}}. \quad \therefore \frac{FL}{T}$

g) $[s]=\text{N}\ /\ \text{m}. \quad \therefore \frac{F}{L}$

6.3 (**A**) The dimensions on the variables are as follows:

$$[\dot{W}]=[F\times V]=M\frac{L}{T^2}\times\frac{L}{T}=\frac{ML^2}{T^3}, \quad [d]=L, \quad [\Delta p]=\frac{ML/T^2}{L^2}=\frac{M}{LT^2}, \quad [V]=\frac{L}{T}$$

First, eliminate T by dividing $\dot{W}$ by Δp. That leaves T in the denominator so divide by V leaving L^2 in the numerator. Then divide by d^2. That provides

$$p=\frac{\dot{W}}{\Delta pVd^2}$$

6.4 $$\underline{\therefore \frac{T}{rw^2R^5}=f_1\left(\frac{e}{R},\ \frac{r}{R},\ \frac{\ell}{R},\ \frac{m}{rwR^2}\right)}.$$

6.5 (**A**) $V = f(d,\ l,g,\ w,\ m)$. The units on the variables on the rhs are as follows:

$$[d] = L,\ \ [l] = L,\ \ [g] = \frac{L}{T^2},\ \ [w] = T^{-1},\ \ [m] = \frac{ML}{T}$$

Because mass M occurs in only one term, it cannot enter the relationship.

6.6 $V = f(\ell, r, m)$. $[V] = \frac{L}{T},\ [\ell] = L,\ [r] = \frac{M}{L^3},\ [m] = \frac{M}{LT}$.

$\therefore$ There is one $p-$term: $p_1 = \frac{rV\ell}{m}$.

$\therefore p_1 = f_1(p_2^0) = \text{Const.}$ $\therefore\ \underline{r\frac{V\ell}{m} = C,}$ or $\text{Re} = \text{Const.}$

6.7 $V = f(s, r, d)$. $[V] = \frac{L}{T},\ \ [s] = \frac{M}{T^2},\ \ [r] = \frac{M}{L^3},\ \ [d] = L.$

$\therefore p_1 = \frac{s}{rV^2 d}$. $\therefore p_1 = f_1(p_2^0) = C\text{onst.}$ $\therefore \underline{\frac{s}{rV^2 d} = C,}$ or We = Const.

6.8 $V = f(H, g, m)$. $[V] = \frac{L}{T},\ \ [g] = \frac{L}{T^2},\ \ [m] = M,\ \ [H] = L.$

$\therefore p_1 = \frac{gHm^0}{V^2}$. $\therefore p_1 = C$. $\therefore \underline{V = \sqrt{gH/C}}$.

6.9 $V = f(H, g, m, r, m)$. $[V] = \frac{L}{T},\ \ [H] = L,\ \ [g] = \frac{L}{T^2},\ \ [m] = M,\ [r] = \frac{M}{L^3},\ \ [m] = \frac{M}{LT}$.

Choose repeating variables H, g, r (select ones with simple dimensions-we couldn't select *V, H,* and *g* since *M* is not contained in any of those terms):

$p_1 = VH^{a_1}g^{b_1}r^{c_1},\ \ p_2 = mH^{a_2}g^{b_2}r^{c_2},\ \ p_3 = mH^{a_3}g^{b_3}r^{c_3}$.

$\therefore p_1 = \frac{Vr^0}{\sqrt{g}\sqrt{H}} = \frac{V}{\sqrt{gH}}.\ \ p_2 = \frac{m}{rH^3}.\ \ p_3 = \frac{m}{r\sqrt{g}H^{3/2}} = \frac{m}{r\sqrt{gH^3}}$.

$$\therefore \underline{\frac{V}{\sqrt{gH}} = f_1\left(\frac{m}{rH^3}, \frac{m}{r\sqrt{gH^3}}\right)}.$$

Note: The above dimensionless groups are formed by observation: simply combine the dimensions so that the $p-$term is dimensionless. We could have set up equations similar to those of Eq. 6.2.11 and solved for a_1, b_1, c_1 and a_2, b_2, c_2 and a_3, b_3, c_3. But the method of observation is usually successful.

6.10 $F_D = f(d, \ell, V, m, r)$. $[F_D] = \frac{ML}{T^2},\ [d] = L,\ [V] = \frac{L}{T},\ [m] = \frac{M}{LT},\ [r] = \frac{M}{L^3}$.

$$p_1 = F_D \ell^{a_1} V^{b_1} r^{c_1}, \quad p_2 = dV^{b_2} r^{c_2} \ell^{a_2}, \quad p_3 = m\ell^{a_3} V^{b_3} r^{c_3}.$$

$$\therefore p_1 = \frac{F_D}{V^2 r\ell^2}, \quad p_2 = \frac{d}{\ell}, \quad p_3 = \frac{m}{rV\ell}.$$

$$\therefore \underline{\frac{F_D}{r\ell^2 V^2} = f_1\left(\frac{d}{\ell}, \frac{m}{r\ell V}\right).}$$

We could write $\frac{p_1}{p_2^2} = f_2\left(\frac{1}{p_2}, \frac{p_3}{p_2}\right)$ or $\frac{F_D}{rd^2V^2} = f_2\left(\frac{\ell}{d}, \frac{m}{rdV}\right)$. This is equivalent to the above. Either functional form must be determined by experimentation.

6.11 $F_D = f(d, \ell, V, m, r)$. $[F_D] = \frac{ML}{T^2}$, $[d] = L$, $[V] = \frac{L}{T}$, $[m] = \frac{M}{LT}$, $[r] = \frac{M}{L^3}$.

$$p_1 = F_D d^{a_1} m^{b_1} V^{c_1}, \quad p_2 = \ell d^{a_2} m^{b_2} V^{c_2}, \quad p_3 = r d^{a_3} m^{b_3} V^{c_3}.$$

By observation we have $p_1 = \frac{F_D}{mVd}, \quad p_2 = \frac{\ell}{d}, \quad p_3 = \frac{rVd}{m}.$

$$\therefore \underline{\frac{F_D}{mVd} = f_1\left(\frac{\ell}{d}, \frac{rVd}{m}\right).}$$

Rather than $p_1 = f_1(p_2, p_3)$, we could write

$\frac{p_1}{p_3} = f_2\left(p_2, \frac{1}{p_3}\right)$, an acceptable form: $\frac{F_D}{rV^2d^2} = f_2\left(\frac{\ell}{d}, \frac{m}{rVd}\right).$

6.12 $h = f(s, d, g, b, g)$. $[h] = L$, $[s] = \frac{M}{T^2}$, $[d] = L$, $[g] = \frac{M}{L^2T^2}$, $[b] = 1$, $[g] = \frac{L}{T^2}$.

Select d, g, g as repeating variables.

$$p_1 = hd^{a_1} g^{b_1} g^{c_1}, \quad p_2 = s d^{a_2} g^{b_2} g^{c_2}, \quad p_3 = b.$$

$$\therefore p_1 = \frac{h}{d}, \quad p_2 = \frac{s}{gd^2}, \quad p_3 = b.$$

$\therefore \underline{\frac{h}{d} = f_1\left(\frac{s}{gd^2}, b\right).}$ Note: gravity does not enter the answer.

6.13 $F_C = f(m, w, R)$. $[F_C] = \frac{ML}{T^2}$, $[m] = M$, $[w] = \frac{1}{T}$, $[R] = L$.

$\therefore p_1 = F_C m^a w^b R^c = \frac{F_C}{mw^2R}. \quad \therefore \frac{F_C}{mw^2R} = C. \quad \therefore \underline{F_C = Cmw^2R}$

6.14 $s = f(M, y, I)$. $[s] = \frac{M}{LT^2}$, $[M] = \frac{ML^2}{T^2}$, $[y] = L$, $[I] = L^4$. $\therefore p_1 = sM^a y^b I^c$.

Given that $b = -1$, $p_1 = \frac{s\, I}{yM} = \text{Const.}$ $\quad \therefore \underline{s = C\frac{My}{I}}.$

6.15 $V = f\left((m, d, \frac{dp}{dx}\right).$ $[V] = \frac{L}{T},\ [m] = \frac{M}{LT},\ [d] = L,\ \left[\frac{dp}{dx}\right] = \frac{M}{L^2T^2}.$

$\therefore p_1 = Vm^a d^b \left(\frac{dp}{dx}\right)^c$. Let's start with the ratio $\frac{m}{dp/dx}$ so that "M" is accounted for.

Then the p_1 – term is $\frac{mV}{dp/dx\, d^2}$. Hence,

$$p_1 = \frac{Vm}{dp/\ dx\ d^2} = \text{Const.} \qquad \therefore \underline{V = \text{Const}\ \frac{d^2 dp/\ dx}{m}}.$$

6.16 $V = f(H, g, r).$ $[V] = \frac{L}{T},\ [H] = L,\ [g] = \frac{L}{T^2},\ [r] = \frac{M}{L^3}.$

$\therefore p_1 = VH^a g^b r^c = V\frac{r^0}{\sqrt{g}\sqrt{H}} = \text{Const.} \qquad \therefore V = \underline{\text{Const.}\ \sqrt{gH}}.$

Density does not enter the expression.

6.17 $V = f(H, m, r, g, d).$ $[V] = \frac{L}{T}, [H] = L, [m] = \frac{M}{LT}, [r] = \frac{M}{L^3}, [g] = \frac{L}{T^2}, [d] = L.$

$p_1 = VH^{a_1} r^{b_1} g^{c_1},\ p_2 = mH^{a_2} r^{b_2} g^{c_2},\ p_3 = dH^{a_3} r^{b_3} g^{c_3}.$ $\left.\begin{matrix}\text{Repeating}\\ \text{variables}\end{matrix}\right\} H, r, g.$

$$p_1 = \frac{V}{\sqrt{gH}},\ p_2 = \frac{m}{r\sqrt{g}H^{3/2}},\ p_3 = \frac{d}{H}.$$

$$\therefore p_1 = f_1(p_2, p_3), \quad \text{or} \quad \underline{\frac{V}{\sqrt{gH}} = f_1\left(\frac{m}{r\sqrt{gH^3}},\ \frac{d}{H}\right)}.$$

6.18 $\Delta p = f(V, d, n, L, e, r).$

$[\Delta p] = \frac{M}{LT^2}, [V] = \frac{L}{T}, [d] = L, [n] = \frac{L^2}{T}, [L] = L, [e] = L, [r] = \frac{M}{L^3}.$

Repeating variables: V, d, r.

$p_1 = \Delta p V^{a_1} d^{b_1} r^{c_1},\ p_2 = n V^{a_2} d^{b_2} r^{c_2},\ p_3 = L V^{a_3} d^{b_3} r^{c_3},\ p_4 = e V^{a_4} d^{b_4} r^{c_4}.$

$$\therefore p_1 = \frac{\Delta p}{rV^2},\ p_2 = \frac{n}{Vd},\ p_3 = \frac{L}{d},\ p_4 = \frac{e}{d}.$$

$$p_1 = f_1(p_2, p_3, p_4). \qquad \therefore \underline{\frac{\Delta p}{rV^2} = f_1\left(\frac{n}{Vd}, \frac{L}{d}, \frac{e}{d}\right)}.$$

6.19 $F_D = f(V, r, m, c, h, r, f, w, a)$ where c is the chord length, h is the maximum thickness, r is the nose radius, f is the trailing edge angle, and a is the angle of attack. Repeating variables: V, c, r. The $p -$ terms are

$$p_1 = \frac{F_D}{rV^2c^2},\ p_2 = \frac{Vrc}{m},\ p_3 = \frac{c}{h},\ p_4 = \frac{c}{r},\ p_5 = f,\ p_6 = \frac{c}{w},\ p_7 = a.$$

Then,

$$\underline{\frac{F_D}{rV^2c^2} = f_1\left(\frac{Vrc}{m}, \frac{c}{h}, \frac{c}{r}, f, \frac{c}{w}, a\right)}$$

6.20 $Q = f(R, A, e, S, g)$. $[Q] = \frac{L^3}{T}, [R] = L, [A] = L^2, [e] = L, [s] = 1, [g] = \frac{L}{T^2}$.

There are only two basic dimensions. Choose two repeating variables, R and g. Then,

$$p_1 = QR^{a_1}g^{b_1},\ p_2 = AR^{a_2}g^{b_2},\ p_3 = eR^{a_3}g^{b_3},\ p_4 = sR^{a_4}g^{b_4}.$$

$$\therefore p_1 = \frac{Q}{\sqrt{g}R^{5/2}},\ p_2 = \frac{A}{R^2},\ p_3 = \frac{e}{R},\ p_4 = s.$$

$$\therefore p_1 = f_1(p_2, p_3, p_4). \quad \therefore \underline{\frac{Q}{\sqrt{gR^5}} = f_1\left(\frac{A}{R^2}, \frac{e}{R}, s\right)}.$$

6.21 $V_p = f(h, g, s, r)$. $[V_p] = \frac{L}{T}, [h] = L, [g] = \frac{L}{T^2}, [s] = \frac{M}{T^2}, [r] = \frac{M}{L^3}$.

Repeating variables: h, r, g. $\therefore p_1 = V_p h^{a_1} r^{b_1} g^{c_1},\ p_2 = s h^{a_2} r^{b_2} g^{c_2}$.

$$\therefore p_1 = \frac{V_p}{\sqrt{hg}},\ p_2 = \frac{s}{rgh^2}. \qquad \therefore \underline{\frac{V_p}{\sqrt{gh}} = f_1\left(\frac{s}{rgh^2}\right)}.$$

6.22 $F_D = f(V, m, r, e, I, d)$. Repeating variables: V, r, d.

$$[F_D] = \frac{ML}{T^2}, [V] = \frac{L}{T}, [m] = \frac{M}{LT}, [r] = \frac{M}{L^3}, [e] = L, [I] = 1, [d] = L.$$

$$p_1 = F_D V^{a_1} r^{b_1} d^{c_1},\ p_2 = mV^{a_2} r^{b_2} d^{c_2},\ p_3 = e\,V^{a_3} r^{b_3} d^{c_3},\ p_4 = I\,V^{a_4} r^{b_4} d^{c_4}.$$

$$\therefore p_1 = \frac{F_D}{rV^2d^2},\ p_2 = \frac{m}{Vrd},\ p_3 = \frac{e}{d},\ p_4 = I.$$

$$\therefore \underline{\frac{F_D}{rV^2d^2} = f_1\left(\frac{m}{Vrd}, \frac{e}{d}, I\right)}.$$

6.23 $F_D = f(V, r_s, r, m, D, g)$. Repeating variables: V, r, D.

$$[F_D] = \frac{ML}{T^2}, [V] = \frac{L}{T}, [r_s] = \frac{M}{L^3}, [r] = \frac{M}{L^3}, [m] = \frac{M}{LT}, [D] = L, [g] = \frac{L}{T^2}.$$

$$p_1 = F_D V^{a_1} r^{b_1} D^{c_1},\ p_2 = r_s V^{a_2} r^{b_2} D^{c_2},\ p_3 = mV^{a_3} r^{b_3} D^{c_3},\ p_4 = gV^{a_4} r^{b_4} D^{c_4}.$$

$$\therefore p_1 = \frac{F_D}{rV^2 D^2},\ p_2 = \frac{r_s}{r},\ p_3 = \frac{m}{rVD},\ p_4 = \frac{gD}{V^2}.$$

$$\therefore \underline{\frac{F_D}{rV^2 D^2} = f_1\left(\frac{r_s}{r}, \frac{m}{rVD}, \frac{gD}{V^2}\right)}.$$

6.24 $F_D = f(V, m, r, d, e, r, c)$. Repeating variables: V, r, d.

$$[F_D] = \frac{ML}{T^2},\ [V] = \frac{L}{T},\ [m] = \frac{M}{LT},\ [r] = \frac{M}{L^3},\ [d] = L,\ [e] = L,\ [r] = L,\ [c] = \frac{1}{L^2}.$$

$$p_1 = F_D V^{a_1} r^{b_1} d^{c_1},\ p_2 = mV^{a_2} r^{b_2} d^{c_2},\ p_3 = eV^{a_3} r^{b_3} d^{c_3},\ p_4 = rV^{a_4} r^{b_4} d^{c_4},\ p_5 = cV^{a_5} r^{b_5} d^{c_5}.$$

$$\therefore p_1 = \frac{F_D}{rV^2 d^2},\ p_2 = \frac{m}{rVd},\ p_3 = \frac{e}{d},\ p_4 = \frac{r}{d},\ p_5 = cd^2.$$

$$\therefore \underline{\frac{F_D}{rV^2 d^2} = f_1\left(\frac{m}{rVd}, \frac{e}{d}, \frac{r}{d}, cd^2\right)}.$$

6.25 $f = g(m, r, V, d)$. $[f] = \frac{1}{T},\ [m] = \frac{M}{LT},\ [r] = \frac{M}{L^3},\ [V] = \frac{L}{T},\ [d] = L.$

Repeating variables, $V,\ d,\ r$. $p_1 = f V^{a_1} d^{b_1} r^{c_1},\ p_2 = mV^{a_2} d^{b_2} r^{c_2}$

$$\therefore p_1 = \frac{fd}{V},\ p_2 = \frac{m}{rVd}. \qquad \therefore \underline{\frac{fd}{V} = g_1\left(\frac{m}{rVd}\right)}.$$

6.26 $F_L = f(V,\ c,\ r,\ \ell_c,\ t,\ a)$. Repeating variables: $V,\ r,\ \ell_c$.

$$[F_L] = \frac{ML}{T^2},\ [V] = \frac{L}{T},\ [c] = \frac{L}{T},\ [r] = \frac{M}{L^3},\ [\ell_c] = L,\ [t] = L,\ [a] = 1.$$

$$p_1 = F_L V^{a_1} r^{b_1} \ell_c^{\,c_1},\ p_2 = cV^{a_2} r^{b_2} \ell_c^{\,c_2},\ p_3 = tV^{a_3} r^{b_3} \ell_c^{\,c_3},\ p_4 = aV^{a_4} r^{b_4} \ell_c^{\,c_4}.$$

$$\therefore p_1 = \frac{F_L}{rV^2 \ell_c^{\,2}},\ p_2 = \frac{c}{V},\ p_3 = \frac{t}{\ell_c},\ p_4 = a.$$

$$\therefore \underline{\frac{F_L}{rV^2 \ell_c^{\,2}} = f_1\left(\frac{c}{V}, \frac{t}{\ell_c}, a\right)}.$$

6.27 $T = f(d,\ w,\ r,\ m,\ t)$. $[T] = \frac{ML^2}{T^2}, [d] = L, [w] = \frac{1}{T}, [r] = \frac{M}{L^3}, [m] = \frac{M}{LT}, [t] = L.$

Repeating variables: $d,\ w,\ r$. $p_1 = Td^{a_1} w^{b_1} r^{c_1},\ p_2 = md^{a_2} w^{b_2} r^{c_2},\ p_3 = t\, d^{a_3} w^{b_3} r^{c_3}$.

$$\therefore p_1 = \frac{T}{rw^2d^5},\ p_2 = \frac{m}{rwd^2},\ p_3 = \frac{t}{d}.$$

$$\therefore \underline{\frac{T}{rw^2d^5} = f_1\left(\frac{m}{rwd^2}, \frac{t}{d}\right)}. \qquad \underline{\dot{W} = rw^3d^5 f_1\left(\frac{m}{rwd^2}, \frac{t}{d}\right)}$$

6.28 $F_D = f(V,\ r,\ m,\ d,\ L,\ r_c,\ w)$ where d is the cable diameter, L the cable length, r_c the cable density, and w the vibration frequency.

Repeating variables: $V,\ d,\ r$. The $p -$ terms are

$$p_1 = \frac{F_D}{rV^2d^2},\ p_2 = \frac{Vdr}{m},\ p_3 = \frac{d}{L},\ p_4 = \frac{r}{r_c},\ p_5 = \frac{V}{wd}$$

We then have

$$\underline{\frac{F_D}{rV^2d^2} = f_1\left(\frac{Vdr}{m},\ \frac{d}{L},\ \frac{r}{r_c},\ \frac{V}{wd}\right)}$$

6.29 $\Delta p = f(D,\ h,\ w,\ r,\ d_1,\ d_0)$. Repeating variables: $D,\ w,\ r$.

$$[\Delta p] = \frac{M}{LT^2},\ [D] = L,\ [h] = L,\ [w] = \frac{1}{T},\ [r] = \frac{M}{L^3},\ [d_1] = L,\ [d_0] = L$$

$$p_1 = \frac{\Delta p}{rw^2D^2},\ p_2 = \frac{h}{D},\ p_3 = \frac{d_1}{D},\ p_4 = \frac{d_0}{D}.$$

$$\therefore \frac{\Delta p}{rw^2D^2} = f_1\left(\frac{h}{D},\ \frac{d_1}{D},\ \frac{d_0}{D}\right). \quad \dot{W} = \text{force} \times \text{velocity} = \Delta p D^2 \times wD.$$

$$\therefore \underline{\dot{W} = rw^3D^5 f_1\left(\frac{h}{D},\ \frac{d_1}{D},\ \frac{d_0}{D}\right)}.$$

6.30 $T = g(f,\ w,\ d,\ H,\ \ell,\ N,\ h,\ r)$. Repeating variables: $w,\ d,\ r$.

$$[T] = \frac{ML^2}{T^2},\ [f] = \frac{1}{T},\ [w] = \frac{1}{T},\ [d] = L,\ [H] = L,\ [\ell] = L,\ [N] = 1,\ [h] = L,\ [r] = \frac{M}{L^3}.$$

$$p_1 = \frac{T}{rw^2d^5},\ p_2 = \frac{f}{w},\ p_3 = \frac{H}{d},\ p_4 = \frac{\ell}{d},\ p_5 = N,\ p_6 = \frac{h}{d}.$$

$$\therefore \underline{\frac{T}{rw^2d^5} = g_1\left(\frac{f}{w},\ \frac{H}{d},\ \frac{\ell}{\mathrm{d}},\ N,\ \frac{h}{d}\right)}.$$

6.31 $Q = f(H,\ w,\ g,\ m,\ r,\ s)$. Repeating variables: $H,\ g,\ r$.

$$[Q] = \frac{L^3}{T},\ [H] = L,\ [w] = L,\ [g] = \frac{L}{T^2},\ [m] = \frac{M}{LT},\ [r] = \frac{M}{L^3},\ [s] = \frac{M}{T^2}.$$

$$\therefore p_1 = \frac{Q}{\sqrt{gH^5}},\ p_2 = \frac{w}{H},\ p_3 = \frac{m}{r\sqrt{gH^3}},\ p_4 = \frac{s}{rgH^2}.$$

$$\underline{\therefore \frac{Q}{\sqrt{gH^5}} = f_1\left(\frac{w}{H},\ \frac{m}{r\sqrt{gH^3}},\ \frac{s}{rgH^2}\right)}.$$

6.32 $d = f(V,\ V_j,\ D,\ s,\ r,\ m,\ r_a)$. Repeating variables: $V_j,\ D,\ r$.

$$[d] = L,\ [V] = \frac{L}{T},\ [V_j] = \frac{L}{T},\ [D] = L,\ [s] = \frac{M}{T^2},\ [r] = \frac{M}{L^3},\ [m] = \frac{M}{LT},\ [r_a] = \frac{M}{L^3}.$$

$$p_1 = \frac{d}{D},\ p_2 = \frac{V}{V_j},\ p_3 = \frac{s}{rV_j^2D},\ p_4 = \frac{m}{rV_jD},\ p_5 = \frac{r_a}{r}.$$

$$\underline{\therefore \frac{d}{D} = f_1\left(\frac{V}{V_j},\ \frac{s}{rV_j^2D},\ \frac{m}{rV_jD},\ \frac{r_a}{r}\right)}.$$

6.33 $T = f(w,\ H,\ h,\ R,\ t,\ m,\ r)$. Repeating variables: $w,\ h,\ r$.

$$[T] = \frac{ML^2}{T^2},\ [w] = \frac{1}{T},\ [H] = L,\ [h] = L,\ [R] = L,\ [t] = L,\ [m] = \frac{M}{LT},\ [r] = \frac{M}{L^3}.$$

$$p_1 = \frac{T}{rw^2d^5},\ p_2 = \frac{H}{h},\ p_3 = \frac{R}{h},\ p_4 = \frac{t}{h},\ p_5 = \frac{m}{rwh^2}$$

$$\underline{\therefore \frac{T}{rw^2d^5} = f_1\left(\frac{H}{h},\ \frac{R}{h},\ \frac{t}{h},\ \frac{m}{rwh^2}\right)}.$$

6.34 $m = f(D,\ H,\ \ell,\ g,\ r,\ V)$. D = tube dia., H = head above outlet, ℓ = tube length.

Repeating variables: $D,\ V,\ r$. $p_1 = \frac{m}{rVD},\ p_2 = \frac{H}{D},\ p_3 = \frac{\ell}{D},\ p_4 = \frac{gD}{V^2}$

$$\underline{\therefore \frac{m}{rVD} = f_1\left(\frac{H}{D},\ \frac{\ell}{D},\ \frac{gD}{V^2}\right)}.$$

6.35 $T = f(R,\ w,\ r,\ e,\ r,\ m,\ \ell)$. Repeating variables: $R, w,\ r$.

$$[T] = \frac{ML^2}{T^2},\ [R] = L,\ [w] = \frac{1}{T},\ [r] = \frac{M}{L^3},\ [e] = L,\ [r] = L,\ [m] = \frac{M}{LT},\ [\ell] = L.$$

$$p_1 = \frac{T}{rw^2R^5},\ p_2 = \frac{e}{R},\ p_3 = \frac{r}{R},\ p_4 = \frac{\ell}{R},\ p_5 = \frac{m}{rwR^2}$$

$$\underline{\therefore \frac{T}{rw^2R^5} = f_1\left(\frac{e}{R},\ \frac{r}{R},\ \frac{\ell}{R},\ \frac{m}{rwR^2}\right)}.$$

6.36 $y_2 = f(V_1,\ y_1,\ r,\ g)$. Neglect viscous wall shear.

$[y_2] = L,\ [V_1] = \frac{L}{T},\ [y_1] = L,\ [r] = \frac{M}{L^3},\ [g] = \frac{L}{T^2}$. Repeating variables: $V_1,\ y_1,\ r$.

$$p_1 = \frac{y_2}{y_1},\ p_2 = \frac{gy_1}{V_1^2}. \qquad (r \text{ does not enter the problem}).$$

$$\therefore \underline{\frac{y_2}{y_1} = f\left(\frac{gy_1}{V_1^2}\right)}.$$

6.37 $f = g(d,\ \ell,\ r,\ m,\ V)$. $[f] = \frac{1}{T},\ [d] = L,\ [\ell] = L,\ [r] = \frac{M}{L^3},\ [m] = \frac{M}{LT},\ [V] = \frac{L}{T}.$

Repeating variables: $d,\ r,\ V$. (ℓ = length of cylinder).

$$p_{1.} = \frac{fd}{V},\ p_2 = \frac{\ell}{d},\ p_3 = \frac{m}{rVd}. \qquad \therefore \underline{\frac{fd}{V} = f_1\left(\frac{\ell}{d},\ \frac{m}{rVd}\right)}.$$

6.38 $\frac{Q_m}{Q_p} = \frac{V_m \ell_m^2}{V_p \ell_p^2},\ \frac{\Delta p_m}{\Delta p_p} = \frac{r_m V_m^2}{r_p V_p^2},\ \frac{(F_p)_m}{(F_p)_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2}$

$\frac{t_m}{t_p} = \frac{r_m V_m^2}{r_p V_p^2},\ \frac{T_m}{T_p} = \frac{r_m V_m^2 \ell_m^3}{r_p V_p^2 \ell_p^3},\ \frac{\dot{Q}_m}{\dot{Q}_p} = \frac{r_m V_m^3 \ell_m^2}{r_p V_p^3 \ell_p^2}$

($\dot{Q}$ has same dimensions as $\dot{W}$.)

6.39 (**A**) $\text{Re}_m = \text{Re}_p$. $\frac{V_m L_m}{\not{n}_m} = \frac{V_p L_p}{\not{n}_p}$. $\therefore V_m = V_p \frac{L_p}{L_m} = 12 \times 9 = 108$ m/s.

6.40 **A**) $\text{Re}_m = \text{Re}_p$. $\frac{V_m L_m}{n_m} = \frac{V_p L_p}{n_p}$. $\therefore V_m = V_p \frac{L_p}{L_m}\frac{n_m}{n_p} = 4 \times 10 \frac{1.51 \times 10^{-5}}{1.31 \times 10^{-6}} = 461$ m/s.

6.41 a) $\text{Re}_m = \text{Re}_p$. $\frac{V_m d_m}{n_m} = \frac{V_p d_p}{n_p}$. $\therefore \frac{V_m}{V_p} = \frac{d_p}{d_m} = 7.$

$\frac{Q_m}{Q_p} = \frac{V_m \ell_m^2}{V_p \ell_p^2}$. $\therefore Q_m = Q_p \frac{V_m}{V_p}\frac{\ell_m^2}{\ell_p^2} = 1.5 \times 7 \times \frac{1}{7^2} = \underline{0.214\ \text{m}^3/\text{s}}.$

$\frac{\dot{W}_m}{\dot{W}_p} = \frac{r_m V_m^3 \ell_m^2}{r_p V_p^3 \ell_p^2} = 7^3 \times \frac{1}{7^2} = 7.$ $\therefore \dot{W}_m = 7 \times 200 = \underline{1400\ \text{kW}}.$

b) $\text{Re}_m = \text{Re}_p$. $\therefore \frac{V_m}{V_p} = \frac{d_p}{d_m}\frac{n_m}{n_p} = 7 \times \frac{.9}{1.3} = 4.85.$

$Q_m = 1.5 \times 4.85 \times \frac{1}{7^2} = \underline{0.148\ \text{m}^3/\text{s}}.$

$\dot{W}_m = 4.85^3 \times \frac{1}{7^2} \times 200 = \underline{466\ \text{kW}}$

6.42 a) $\text{Re}_m = \text{Re}_p. \quad \frac{V_m d_m}{n_m} = \frac{V_p d_p}{n_p}. \quad \therefore \frac{V_m}{V_p} = \frac{d_p}{d_m} = 5.$

$$\frac{\dot{m}_m}{\dot{m}_p} = \frac{r_m \ell_m^2 V_m}{r_p \ell_p^2 V_p} = \frac{1}{5^2} \times 5. \quad \therefore \dot{m}_m = \dot{m}_p \frac{1}{5} = 800/5 = \underline{160 \text{ kg/s}}.$$

$$\frac{\Delta p_m}{\Delta p_p} = \frac{r_m V_m^2}{r_p V_p^2} = 5^2. \quad \therefore \Delta p_m = 25 \Delta p_p = 25 \times 600 = \underline{15\,000 \text{ kPa}}.$$

b) $\text{Re}_m = \text{Re}_p. \quad \therefore \frac{V_m}{V_p} = \frac{d_p}{d_m}\frac{n_m}{n_p} = 5 \times \frac{.8}{1.14} = 3.51.$

$$\dot{m}_m = 800 \times \frac{1}{5^2} \times 3.51 = \underline{112 \text{ kg/s}}. \qquad \Delta p_m = 600 \times 3.51^2 = \underline{7390 \text{ kPa}}.$$

6.43 a) $\text{Re}_m = \text{Re}_p. \quad \frac{V_m d_m}{n_m} = \frac{V_p d_p}{n_p}. \quad \therefore \frac{V_m}{V_p} = \frac{d_p}{d_m} = 10.$

$$\frac{F_m}{F_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2} = 10^2 \times \frac{1}{10^2} = 1. \qquad \therefore F_m = F_p = \underline{10 \text{ lb}}.$$

b) $\text{Re}_m = \text{Re}_p. \quad \therefore \frac{V_m}{V_p} = \frac{d_p}{d_m}\frac{n_m}{n_p} = 10 \times \frac{1.06}{1.41} = 7.52.$

$$F_p = F_m \frac{\cancel{r_p}}{\cancel{r_m}} \frac{V_p^2}{V_m^2} \frac{L_p^2}{L_m^2} = 10 \times \frac{1}{7.52^2} \times 10^2 = \underline{17.68 \text{ lb}}.$$

6.44 $\text{Re}_m = \text{Re}_p. \quad \frac{V_m \ell_m}{n_m} = \frac{V_p \ell_p}{n_p}. \quad \therefore \frac{V_m}{V_p} = \frac{\ell_p}{\ell_m}\frac{n_m}{n_p} = 10 \text{ assuming } \frac{n_m}{n_p} = 1.$

$$\therefore V_m = 10 V_p = 1000 \text{ km / hr}.$$

This velocity is much too high for a model test; it is in the compressibility region. Thus, small-scale models of autos are not used. Full-scale wind tunnels are common.

6.45 $\text{Re}_m = \text{Re}_p. \quad \therefore \frac{V_m \ell_m}{n_m} = \frac{V_p \ell_p}{n_p}. \quad \therefore \frac{V_m}{V_p} = \frac{\ell_p}{\ell_m}\frac{n_m}{n_p}.$

Water: $\frac{V_m}{V_p} = \frac{\ell_p}{\ell_m} = 10 \text{ assuming } n_m = n_p. \quad \therefore V_m = 10 V_p = 900 \text{ km / hr}.$

Air: $V_m = V_p \frac{\ell_p}{\ell_m}\frac{n_m}{n_p} = 90 \times 10 \frac{1.5 \times 10^{-5}}{1 \times 10^{-6}} = 13\,500 \text{ km / hr}.$

Neither a water channel or a wind tunnel is recommended. Full-scale testing in a water channel is suggested.

6.46 $\text{Re}_m = \text{Re}_p$. $\dfrac{V_m \ell_m}{n_m} = \dfrac{V_p \ell_p}{n_p}$. $\therefore V_m / V_p = \ell_p / \ell_m = 10$ if $n_m = n_p$.

$$\therefore V_m = 10 \times 50 = \underline{500 \text{ m/s}}.$$

This is in the compressibility range so is not recommended. Try a water channel for the model study. Then

$$\frac{V_m}{V_p} = \frac{\ell_p}{\ell_m}\frac{n_m}{n_p} = 10 \times \frac{1\times 10^{-6}}{1.5\times 10^{-5}} = 0.662. \qquad \therefore V_m = \underline{33.1 \text{ m/s}}.$$

This is a possibility, although 33.1 m/s is still quite large.

$$\frac{(F_D)_m}{(F_D)_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2} = \frac{1000}{1.23} \times 0.662^2 \times \frac{1}{10^2} = \underline{3.56}.$$

6.47 $\text{Re}_m = \text{Re}_p$. $\dfrac{V_m d_m}{n_m} = \dfrac{V_p d_p}{n_p}$. $\therefore d_m = d_p \dfrac{V_p}{V_m}\dfrac{n_m}{n_p} = 2.5 \times 1 \times \dfrac{1.06\times 10^{-5}}{5.5\times 10^{-3}} = \underline{0.0048 \text{ ft}}$.

Find n_{oil} using Fig. B.2. Then

$$\frac{\Delta p_m}{\Delta p_p} = \frac{r_m V_m^2}{r_p V_p^2} = \frac{1.94}{1.94 \times 0.9} \times 1^2 = \underline{1.11}.$$

6.48 $\text{Re}_m = \text{Re}_p$. $\dfrac{V_m \ell_m}{n_m} = \dfrac{V_p \ell_p}{n_p}$. $\therefore V_m = V_p \dfrac{\ell_p}{\ell_m}\dfrac{n_m}{n_p} = 0.1 \times .025 \times 10^{-3} \times \dfrac{\ell_p}{\ell_m}$.

If $\ell_p \cong 5$ cm, then $\dfrac{\ell_p}{\ell_m} = \dfrac{5}{.0025} = 2000$ and $V_m = 0.005$ m / s.

We could try $\ell_p \cong 50$ cm, but $V_m = 0.05$ m / s. Each of these V_m's is quite small — too small for easy measurements. Let's try a wind tunnel. Then,

$$V_m = V_p \frac{\ell_p}{\ell_m}\frac{n_m}{n_p} = 0.1 \times .025 \times 10^{-3} \frac{\ell_p}{\ell_m} \times \frac{1\times 10^{-3}}{1.8 \times 10^{-5}} = 0.28 \text{ m/s if } \ell_p = 5 \text{ cm. Or,}$$

if $\ell_p = 50$ cm, $V_m = 2.8$ m / s. This is a much better velocity to work with in the lab. Thus, choose a wind tunnel.

6.49 $\text{Re}_m = \text{Re}_p$. $\therefore \dfrac{V_m \ell_m}{n_m} = \dfrac{V_p \ell_p}{n_p}$. $\text{Fr}_m = \text{Fr}_p$. $\therefore \dfrac{V_m^2}{\ell_m g_m} = \dfrac{V_p^2}{\ell_p g_p}$. $\therefore \dfrac{V_m}{V_p} = \sqrt{\dfrac{1}{30}}$.

$$\frac{V_m}{V_p} = \frac{\ell_p}{\ell_m}\frac{n_m}{n_p} = 30\frac{n_m}{n_p} = \sqrt{\frac{1}{30}}. \quad \therefore \frac{n_m}{n_p} = \frac{1}{164}. \quad \therefore n_m = \underline{6.1\times 10^{-9} \text{ m}^2 / \text{s}}. \text{ Impossible!}$$

6.50 (**C**) $\text{Fr}_m = \text{Fr}_p$. $\dfrac{V_m^2}{l_m g_m} = \dfrac{V_p^2}{l_p g_p}$. $\therefore V_m = V_p \sqrt{\dfrac{l_m}{l_p}} = 2 \times \dfrac{1}{4} = 0.5$ m/s.

6.51 (**A**) From Froude's number $V_m = V_p \sqrt{\frac{l_m}{l_p}}$. From the dimensionless force we have:

$$F_m^* = F_p^* \quad \text{or} \quad \frac{F_m}{r_m V_m^2 l_m^2} = \frac{F_p}{r_p V_p^2 l_p^2}. \quad \therefore F_p = F_m \frac{V_p^2}{V_m^2} \frac{l_p^2}{l_m^2} = 10 \times 25 \times 25^2 = 156000 \text{ N}.$$

6.52 $\text{Fr}_m = \text{Fr}_p. \quad \therefore \frac{V_m^2}{\ell_m g_m} = \frac{V_p^2}{\ell_p g_p}. \quad \therefore V_m = V_p \sqrt{\frac{\ell_m}{\ell_p}} = 10\sqrt{\frac{1}{60}} = \underline{1.29 \text{ m / s}}.$

$$\frac{(F_D)_m}{(F_D)_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2}. \quad \therefore (F_D)_p = \frac{V_p^2}{V_m^2} \times \frac{\ell_p^2}{\ell_m^2} (F_D)_m = 60 \times 60^2 \times 10 = \underline{2.16 \times 10^6 \text{ N}}.$$

6.53 $\text{Fr}_m = \text{Fr}_p. \quad \frac{V_m^2}{\ell_m g_m} = \frac{V_p^2}{\ell_p g_p}. \quad \therefore \frac{V_m}{V_p} = \sqrt{\frac{\ell_m}{\ell_p}}.$

a) $\frac{Q_m}{Q_p} = \frac{V_m \ell_m^2}{V_p \ell_p^2}. \quad \therefore Q_m = Q_p \frac{V_m}{V_p} \frac{\ell_m^2}{\ell_p^2} = 2 \times \frac{1}{\sqrt{10}} \times \frac{1}{10^2} = \underline{0.00632 \text{ m}^3 \text{ / s}}.$

b) $\frac{F_m}{F_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2}. \quad \therefore F_p = F_m \frac{V_p^2}{V_m^2} \frac{\ell_p^2}{\ell_m^2} = 12 \times 10 \times 10^2 = \underline{12\ 000 \text{ N}}.$

6.54 Neglect viscous effects. $\text{Fr}_m = \text{Fr}_p. \quad \therefore \frac{V_m}{V_p} = \sqrt{\frac{\ell_m}{\ell_p}} = \sqrt{\frac{1}{10}}. \quad \therefore V_p = \underline{63.2 \text{ fps}}.$

$$\frac{F_m}{F_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2}. \quad \therefore F_p = F_m \frac{V_p^2}{V_m^2} \frac{\ell_p^2}{\ell_m^2} = 0.8 \times 10 \times 10^2 = \underline{800 \text{ lb}}.$$

6.55 Neglect viscous effects, and account for wave (gravity) effects.

$$\text{Fr}_m = \text{Fr}_p. \quad \therefore \frac{V_m^2}{\ell_m g_m} = \frac{V_p^2}{\ell_p g_p}. \quad \therefore \frac{V_m}{V_p} = \sqrt{\frac{\ell_m}{\ell_p}}. \quad \frac{w_m}{w_p} = \frac{V_m / \ell_m}{V_p / \ell_p}.$$

$$\therefore w_m = w_p \frac{V_m}{V_p} \frac{\ell_p}{\ell_m} = 600 \times \sqrt{\frac{1}{10}} \times 10 = 1897 \text{ rpm}.$$

$$\frac{T_m}{T_p} = \frac{r_m V_m^2 \ell_m^3}{r_p V_p^2 \ell_p^3}. \quad \therefore T_p = T_m \frac{V_p^2}{V_m^2} \frac{\ell_p^3}{\ell_m^3} = 1.2 \times 10 \times 10^3 = \underline{120\ 000 \text{ N} \cdot \text{m}}.$$

6.56 $\text{Fr}_m = \text{Fr}_p. \quad \therefore \frac{V_m^2}{\ell_m g_m} = \frac{V_p^2}{\ell_p g_p}. \quad \therefore \frac{V_m}{V_p} = \sqrt{\frac{\ell_m}{\ell_p}}. \quad \frac{6}{100} = \sqrt{\frac{\ell_m}{\ell_p}}. \quad \therefore \frac{\ell_p}{\ell_m} = \underline{278}.$

6.57 Check the Reynolds number:

$$\text{Re}_p = \frac{V_p d_p}{\nu_p} = \frac{15 \times 2}{10^{-6}} = 30 \times 10^6.$$

This is a high-Reynolds-number flow.

$$\text{Re}_m = \frac{2 \times 2\ /\ 30}{10^{-6}} = 1.33 \times 10^5.$$

This may be sufficiently large for similarity. If so,

$$\frac{\dot{W}_m}{\dot{W}_p} = \frac{\rho_m V_m^3 \ell_m^2}{\rho_p V_p^3 \ell_p^2} = \frac{2^3}{15^3} \times \frac{1}{30^2} = 2.63 \times 10^{-6}.$$

$$\therefore \dot{W}_p = (2 \times 2.15)\ /\ 2.63 \times 10^{-6} = \underline{1633\ \text{kW}}.$$

6.58 This is due to the separated flow downwind of the stacks, a viscous effect.

$\therefore$ Re is the significant parameter. $\text{Re}_p = \dfrac{10 \times 4}{1.5 \times 10^{-5}} = 26.7 \times 10^5$. This is a high-Reynolds-number flow. Let's assume the flow to be Reynolds number independent above $\text{Re} = 5 \times 10^5$ (see Fig. 6.4). Then

$$\text{Re}_m = 5 \times 10^5 = \frac{V_m \times 4/20}{1.5 \times 10^{-5}}. \qquad \therefore V_m \ \underline{\geq 37.5\ \text{m/s}}.$$

6.59 $\text{Re}_p = \dfrac{20 \times 10}{1.5 \times 10^{-5}} = 13.3 \times 10^6$. This is a high-Reynolds-number flow.

Let $\text{Re}_m = 10^5 = \dfrac{V_m \times .4}{1.5 \times 10^{-5}}$. $\quad \therefore V_m \geq 3.75\ \text{m / s}$ for the wind tunnel.

$\text{Re}_m = 10^5 = \dfrac{V_m \times .1}{1 \times 10^{-6}}$. $\quad \therefore V_m \geq 1.0\ \text{m / s}$ for the water channel.

Either could be selected. The better facility would be chosen.

$$\frac{F_{m_1}}{F_{m_2}} = \frac{\rho_{m_1} V_{m_1}^2 \ell_{m_1}^2}{\rho_{m_2} V_{m_2}^2 \ell_{m_2}^2} = \frac{3.2}{F_{m_2}}. \qquad \therefore F_{m_2} = 3.2 \frac{1000}{1.23} \frac{2.4^2}{15^2} \times \frac{.1^2}{.4^2} = \underline{4.16\ \text{N}}.$$

$$\frac{\dot{W}_m}{\dot{W}_p} = \frac{\rho_m V_m^3 \ell_m^2}{\rho_p V_p^3 \ell_p^2} = \frac{15^3 \times .4^2}{20^3 \times 10^2}. \qquad \therefore \dot{W}_p = (15 \times 3.2) \frac{20^3}{15^3} \times \frac{10^2}{.4^2} = \underline{71\ 100\ \text{W}}.$$

6.60 Re is the significant parameter. This is undoubtedly a high-Reynolds-number flow. If the model is 4' high then $\dfrac{\ell_p}{\ell_m} = 250$, and the model's diameter is 45/250 = 0.18'. For $\text{Re}_m = 3 \times 10^5$, we have

$$\text{Re}_m = 3 \times 10^5 = \frac{V_m \times .18}{1.5 \times 10^{-4}}. \qquad \therefore V_m \geq 250\ \text{fps, and } \underline{\text{a study is possible}}.$$

6.61 Mach No. is the significant parameter. $\text{M}_m = \text{M}_p$.

a) $\text{M}_m = \text{M}_p$. $\quad \therefore \frac{V_m}{c_m} = \frac{V_p}{c_p}$. $\quad \therefore V_m = V_p = \underline{200 \text{ m/s}}$.

$\frac{F_m}{F_p} = \frac{r_m V_m^2 \ell_m^2}{r_p V_p^2 \ell_p^2}$. $\quad \therefore F_p = 10 \times 1^2 \times 20^2 = \underline{4000 \text{ N}}$.

b) $V_p = V_m \frac{c_p}{c_m} = V_m \sqrt{\frac{T_p}{T_m}} = 200\sqrt{\frac{255.7}{296}} = \underline{186 \text{ m/s}}$.

$F_p = F_m \frac{\rho_m V_p^2 \ell_p^2}{\rho_m V_m^2 \ell_m^2} = 10 \times .601 \times \frac{186^2}{200^2} \times 20^2 = \underline{2080 \text{ N}}$.

c) $V_p = V_m \frac{c_p}{c_m} = V_m \sqrt{\frac{T_p}{T_m}} = 200\sqrt{\frac{223.3}{296}} = \underline{174 \text{ m/s}}$.

$F_p = F_m \frac{\rho_p}{\rho_m} \frac{V_p^2}{V_m^2} \frac{\ell_p^2}{\ell_m^2} = 10 \times .338 \times \frac{174^2}{200^2} \times 20^2 = \underline{1023 \text{ N}}$.

6.62 $\text{M}_m = \text{M}_p$. $\quad \therefore \frac{V_m}{c_m} = \frac{V_p}{c_p}$. $\quad \therefore V_m = 250\sqrt{\frac{273}{223.3}} = \underline{276 \text{ m/s}}$.

$\frac{V_m}{V_p} = \frac{c_m}{c_p} = \sqrt{\frac{T_m}{T_p}}$. $\quad \therefore V_p = 290\sqrt{\frac{223.3}{273}} = \underline{262 \text{ m/s}}$.

$\frac{p_m}{p_p} = \frac{r_m V_m^2}{r_p V_p^2}$. $\quad \therefore p_p = p_m \frac{r_p}{r_m} \frac{V_p^2}{V_m^2} = 80 \frac{.338 r_o}{.8 r_o} \frac{262^2}{290^2} = \underline{34.6 \text{ kPa, abs.}}$

$a_p = \underline{5^\circ}$ for similarity. (Note: we use r_m at 2700 m where T = 0°C.)

6.63 a) $\text{Fr}_m = \text{Fr}_p$. $\quad \frac{V_m^2}{\ell_m g_m} = \frac{V_p^2}{\ell_p g_p}$. $\quad \therefore \frac{V_m}{V_p} = \sqrt{\frac{\ell_m}{\ell_p}}$.

$\frac{w_m}{w_p} = \frac{V_m}{V_p} \frac{\ell_p}{\ell_m} = \frac{1}{\sqrt{10}} \times 10$. $\quad \therefore w_m = 2000 \times \frac{10}{\sqrt{10}} = \underline{6320 \text{ rpm}}$.

b) $\text{Re}_m = \text{Re}_p$. $\quad \frac{V_m \ell_m}{n_m} = \frac{V_p \ell_p}{n_p}$. $\quad \therefore \frac{V_m}{V_p} = \frac{\ell_p}{\ell_m} = 10$.

$\frac{w_m}{w_p} = \frac{V_m}{V_p} \frac{\ell_p}{\ell_m} = 10 \times \frac{1}{10} = 1$. $\quad \therefore w_m = \underline{2000 \text{ rpm}}$.

6.64 There are no gravity effects nor compressibility effects. It is a high-Re flow. $\frac{T_m}{T_p} = \frac{r_m V_m^2 \ell_m^3}{r_p V_p^2 \ell_m^3}$. $\quad \therefore T_p = T_m \frac{V_p^2}{V_m^2} \frac{\ell_p^3}{\ell_m^3} = 12 \times \frac{15^2}{60^2} \times 10^3 = \underline{750 \text{ N} \cdot \text{m}}$.

$$\frac{w_m}{w_p} = \frac{V_m}{V_p}\frac{\ell_p}{\ell_m}. \qquad \therefore w_p = w_m \frac{V_p}{V_m}\frac{\ell_m}{\ell_p} = 500 \times \frac{15}{60} \times \frac{1}{10} = \underline{12.5 \text{ rpm}}.$$

6.65 $\text{Re}_m = \text{Re}_p. \qquad \therefore \frac{V_m \ell_m}{n_m} = \frac{V_p \ell_p}{n_p}. \qquad \therefore V_m = V_p \frac{\ell_p}{\ell_m} = 10 \times 10 = 100 \text{ m / s}.$

This is too large for a water channel. Undoubtedly this is a high-Re flow. Select a speed of 5 m/s. For this speed,

$\text{Re}_m = \dfrac{5 \times 0.1}{1 \times 10^{-6}} = 5 \times 10^5$, where we used $\ell_m = 0.1$ ($\ell_p = 1$ m, i.e., the

dia. of the porpoise). $w_m = w_p \dfrac{V_m}{V_p}\dfrac{\ell_p}{\ell_m} = 1 \times \dfrac{5}{10} \times 10 = \underline{5 \text{ motions / second}}.$

6.66 $r^* = \dfrac{r}{r_o},\ t^* = tf,\ u^* = \dfrac{u}{V},\ v^* = \dfrac{v}{V},\ x^* = \dfrac{x}{\ell},\ y^* = \dfrac{y}{\ell}.$ Substitute in:

$$f\ r_o \frac{\P r^*}{\P t^*} + r_o \frac{V}{\ell}\frac{\P (r^* u^*)}{\P x^*} + r_o \frac{V}{\ell}\frac{\P (r^* v^*)}{\P y^*} = 0.$$

Divide by $r_o V / \ell$:

$$\therefore \frac{f\ell}{V}\frac{\P r^*}{\P t^*} + \frac{\P}{\P x^*}(r^* u^*) + \frac{\P}{\P y^*}(r^* v^*) = 0. \qquad \text{parameter} = \underline{\frac{f\ell}{V}}.$$

6.67 $\vec{V}^* = \dfrac{\vec{V}}{U},\ u^* = \dfrac{u}{U},\ v^* = \dfrac{v}{U},\ w^* = \dfrac{w}{U},\ x^* = \dfrac{x}{\ell},\ \ y^* = \dfrac{y}{\ell},\ \ z^* = \dfrac{z}{\ell},\ p^* = \dfrac{p}{rU^2},\ \ t^* = tf.$

Substitute into Euler's equation and obtain:

$$Uf \frac{\P \vec{V}^*}{\P t^*} + \frac{U^2}{\ell} u^* \frac{\P \vec{V}^*}{\P x^*} + \frac{U^2}{\ell} v^* \frac{\P \vec{V}^*}{\P y^*} + \frac{U^2}{\ell} w^* \frac{\P \vec{V}^*}{\P z^*} = -\frac{rU^2}{\ell}\frac{\vec{\nabla}^* p^*}{r}.$$

Divide by U^2 / ℓ:

$$\frac{f\ell}{U}\frac{\partial \vec{V}^*}{\partial t^*} + u^* \frac{\partial \vec{V}^*}{\partial x^*} + v^* \frac{\partial \vec{V}^*}{\partial y^*} + w^* \frac{\partial \vec{V}^*}{\partial z^*} = -\nabla^* p^*. \qquad \text{Parameter} = \underline{\frac{f\ell}{U}}$$

6.68 $\vec{V}^* = \dfrac{\vec{V}}{U},\ t^* = \dfrac{tU}{\ell},\ \vec{\nabla}^* = \ell\vec{\nabla},\ p^* = \dfrac{p}{rU^2},\ h^* = \dfrac{h}{\ell}.$ Euler's equation is then

$$r\frac{U^2}{\ell}\frac{D\vec{V}^*}{Dt^*} = -r\frac{U^2}{\ell}\vec{\nabla}^* p^* - rg\frac{\ell}{\ell}\vec{\nabla}^* h^*.$$

Divide by rU^2 / ℓ:

$$\frac{D\vec{V}^*}{Dt^*} = -\vec{\nabla}^* p^* - \frac{g\ell}{U^2}\vec{\nabla}^* h^*. \qquad \text{Parameter} = \underline{\frac{g\ell}{U^2}}.$$

6.69 There is no y- or z-component velocity so continuity requires that $\P u / \P x = 0$. There is no initial pressure distribution tending to cause motion so $\P p / \P x = 0$. The

x-component Navier-Stokes equation is then

$$\frac{\partial u}{\partial t}+u\cancel{\frac{\partial u}{\partial x}}+\cancel{v}\frac{\partial u}{\partial y}+\cancel{w}\frac{\partial u}{\partial z}=-\frac{1}{\rho}\cancel{\frac{\partial p}{\partial x}}+g_x+\nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}+\cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

(wide plates)

This simplifies to

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}.$$

a) Let $u^*=u/U,\ y^*=y/h$ and $t^*=tU/h$. Then

$$\frac{U^2}{h}\frac{\partial u^*}{\partial t^*}=\frac{\nu U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The normalized equation is

$$\frac{\partial u^*}{\partial t^*}=\frac{1}{\mathrm{Re}}\frac{\partial^2 u^*}{\partial y^{*2}}\qquad \text{where } \mathrm{Re}=\frac{Uh}{\nu}$$

b) Let $u^*=u/U,\ y^*=y/h$ and $t^*=t\nu/h^2$. Then

$$\frac{\nu U}{h^2}\frac{\partial u^*}{\partial t^*}=\nu\frac{U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The normalized equation is

$$\frac{\partial u^*}{\partial t^*}=\frac{\partial^2 u^*}{\partial y^{*2}}$$

6.70 The only velocity component is u. Continuity then requires that $\partial u/\partial x=0$ (replace z with x and v_z with u in the equations written using cylindrical coordinates). The x-component Navier-Stokes equation is

$$\frac{\partial u}{\partial t}+\cancel{v_r}\frac{\partial u}{\partial r}+\frac{\cancel{v_\theta}}{r}\frac{\partial u}{\partial \theta}+u\cancel{\frac{\partial u}{\partial x}}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\cancel{g_x}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\cancel{\frac{\partial^2 u}{\partial \theta^2}}+\cancel{\frac{\partial^2 u}{\partial x^2}}\right)$$

This simplifies to

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)$$

a) Let $u^*=u/V,\ x^*=x/d,\ t^*=tV/d,\ p^*=p/\rho V^2$ and $r^*=r/d$:

$$\frac{V^2}{d}\frac{\partial u^*}{\partial t^*}=-\frac{\rho V^2}{\rho d}\frac{\partial p^*}{\partial x^*}+\frac{\nu V}{d^2}\left(\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

The normalized equation is

$$\frac{\partial u^*}{\partial t^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)\quad \text{where } \mathrm{Re}=\frac{Vd}{\nu}$$

b) Let $u^*=u/V,\ x^*=x/d,\ t^*=t\nu/d^2,\ p^*=p/\rho V^2$ and $r^*=r/d$:

$$\frac{\nu V}{d^2}\frac{\partial u^*}{\partial t^*}=-\frac{\rho V^2}{\rho d}\frac{\partial p^*}{\partial x^*}+\frac{\nu V}{d^2}\left(\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

The normalized equation is

$$\frac{\partial u^*}{\partial t^*} = -\mathrm{Re}\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*} \qquad \text{where } \mathrm{Re} = \frac{Vd}{\nu}$$

6.71 Assume $w = 0$ and $\frac{\partial}{\partial z} = 0$. The x-component Navier-Stokes equation is then

$$\cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + \cancel{v}\frac{\partial u}{\partial y} + w\cancel{\frac{\partial u}{\partial z}} = -\frac{1}{\rho}\cancel{\frac{\partial p}{\partial x}} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

With $g_x = g$ the simplified equation is

$$u\frac{\partial u}{\partial x} = g + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Let $u^* = u/V$, $x^* = x/h$ and $y^* = y/h$. Then

$$\frac{V^2}{h}u^*\frac{\partial u^*}{\partial x^*} = g + \nu\frac{V}{h^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

The normalized equation is

$$u^*\frac{\partial u^*}{\partial x^*} = \frac{1}{\mathrm{Fr}^2} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) \quad \text{where } \mathrm{Fr} = \frac{V}{\sqrt{hg}} \text{ and } \mathrm{Re} = \frac{Vh}{\nu}$$

6.72 $u^* = \frac{u}{U},\quad v^* = \frac{v}{U},\quad T^* = \frac{T}{T_o},\quad x^* = \frac{x}{\ell},\quad y^* = \frac{y}{\ell},\quad \nabla^{*2} = \ell^2\nabla^2.$

$$\rho c_p\left[\frac{UT_o}{\ell}\frac{\partial T^*}{\partial x^*} + \frac{UT_o}{\ell}\frac{\partial T^*}{\partial y^*}\right] = \frac{K}{\ell^2}T_o\nabla^{*2}T^*.$$

Divide by $\rho c_p U T_o/\ell$:

$$\frac{\partial T^*}{\partial x^*} + \frac{\partial T^*}{\partial y^*} = \frac{K}{\rho c_p U\ell}\nabla^{*2}T^*. \qquad \text{Parameter} = \frac{K}{\mu c_p}\frac{\mu}{\rho U\ell} = \underline{\frac{1}{\mathrm{Pr}}\frac{1}{\mathrm{Re}}}.$$

6.73 $\rho^* = \frac{\rho}{\rho_o}, \vec{V}^* = \frac{\vec{V}}{U}, t^* = \frac{tU}{\ell}, \vec{\nabla}^* = \frac{1}{\ell}\vec{\nabla}, \nabla^{*2} = \frac{1}{\ell^2}\nabla^2, p^* = \frac{p}{p_o}, T^* = \frac{T}{T_o}.$

momentum: $$\rho_o\rho^*\frac{U^2}{\ell}\frac{D\vec{V}^*}{Dt^*} = -\frac{p_o}{\ell}\vec{\nabla}^*p^* + \frac{\mu U}{\ell^2}\nabla^{*2}\vec{V}^* + \frac{\mu U}{3\ell^2}\vec{\nabla}^*(\vec{\nabla}^*\cdot\vec{V}^*).$$

Divide by $\rho_o U^2/\ell$:

$$\rho^*\frac{D\vec{V}^*}{Dt^*} = -\frac{p_o}{\rho_o U^2}\vec{\nabla}^*p^* + \frac{\mu}{\rho_o U\ell}\left[\nabla^{*2}\vec{V}^* + \vec{\nabla}^*(\vec{\nabla}^*\cdot\vec{V}^*)\right].$$

energy: $$\rho^* c_v\rho_o T_o\frac{U}{\ell}\frac{DT^*}{Dt^*} = \frac{K}{\ell^2}T_o\nabla^{*2}T^* - p_o\frac{U}{\ell}p^*\vec{\nabla}^*\cdot\vec{V}^*.$$

Divide by $\rho_o c_v T_o U/\ell$:

$$\rho^*\frac{DT^*}{Dt^*} = \frac{K}{\rho_o c_v U\ell}\nabla^{*2}T^* - \frac{p_o}{\rho_o c_v T_o}p^*\vec{\nabla}^*\cdot\vec{V}^*.$$

The parameters are: $\dfrac{p_o}{r_o U^2} = \dfrac{RT_o}{U^2} = \dfrac{kRT_o}{kU^2} = \dfrac{c^2}{kU^2} = \dfrac{1}{k\mathrm{M}^2}$.

$$\frac{m}{r_o U\ell} = \frac{1}{\mathrm{Re}}. \qquad \frac{K}{r_o c_v U\ell} = \frac{K}{mc_p}\frac{c_p}{c_v}\frac{m}{r_o U\ell} = \frac{K}{\mathrm{Pr}\ \mathrm{Re}}.$$

$$\frac{p_o}{r_o c_v T_o} = \frac{RT_o}{c_v T_o} = \frac{c_p - c_v}{c_v} = K - 1.$$

The significant parameters are $\underline{K,\ \mathrm{M},\ \mathrm{Re},\ \mathrm{Pr}}$.